

# Holography and Variable Cosmological Constant

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## Abstract

An effective local quantum field theory with UV and IR cutoffs correlated in accordance with holographic entropy bounds is capable of rendering the cosmological constant (CC) stable against quantum corrections. By setting an IR cutoff to length scales relevant to cosmology, one easily obtains the currently observed  $\rho_\Lambda \simeq 10^{-47} \text{ GeV}^4$ , thus alleviating the CC problem. It is argued that scaling behavior of the CC in these scenarios implies an interaction of the CC with matter sector or a time-dependent gravitational constant, to accommodate the observational data.

It has been pointed out [1, 2, 3, 4] that gravitational holography might provide a natural solution to the CC problem [5]. This follows primarily from the holographic principle [6] which states that the number of independent degrees of freedom residing inside the relevant region is bounded by the surface area in Planck units, instead of by the volume of the region. The principle stems from holographic entropy bounds [7, 8, 9] whose idea historically emerged from the study of black hole entropy and partially from string theory. Such bounds establish black holes as maximally entropic objects of a given size, and postulate that the maximum entropy inside the relevant region behaves non-extensively, growing only as its surface area.

In conventional quantum field theories the CC is not stable against quantum corrections as there the entropy in a region of size  $L$  scales extensively,  $S \sim L^3$ . Taken as a fundamental property of the microscopic theory of quantum gravity, one can use holography to treat the CC problem. One notes that application of the entropy bound [9] to effective field theories,

$$L^3 M^3 \lesssim S_{BH} \sim L^2 M_P^2, \quad (1)$$

does actually suggest that an effective field theory with an arbitrary UV cutoff  $M$  must break down in an arbitrary large volume. Here the size of the system  $L$  acts as an IR cutoff and  $M_P$  is the Planck mass. Cohen et al. [2] strengthened the bound (1) considerably by claiming that conventional quantum field theory actually fails in a much smaller volume. By excluding those states of the system that already have collapsed to a black hole, they arrived at a much stringent limit

$$L^3 M^4 \lesssim L M_P^2. \quad (2)$$

Thus, an effective local quantum field theory can be viable as a good approximate description of physics if and only if UV and IR cutoffs are correlated as in (2).

One immediate implication of Eq. (2) may be seen by calculating the effective CC generated by vacuum fluctuations (zero point energies)

$$\begin{aligned} \rho_\Lambda^{ZPE} &\propto \int_{L^{-1}}^M k^2 dk \sqrt{k^2 + m^2} \sim M^4 & M \gtrsim m \\ &\sim mM^3 & M \lesssim m, \end{aligned} \quad (3)$$

since clearly  $\rho_\Lambda^{ZPE}$  is dominated by UV modes. In both cases (3) the saturated form of Eq.

(2) can be rewritten as

$$\rho_\Lambda(L) \simeq L^{-2} G_N^{-1}(L) , \quad (4)$$

where dependence on the IR cutoff has been made explicit not only in  $\rho_\Lambda$  but also in the Newton's constant,  $G_N \equiv M_P^{-2}$ .<sup>1</sup> Thus Eq. (4) promotes both  $\rho_\Lambda$  and  $G_N$  as dynamical quantities. Specifying  $L$  to be the size of the present Hubble distance ( $L = H_0^{-1} \simeq 10^{28}$  cm) one immediately arrives at the observed value for the dark energy density today  $\rho_\Lambda \simeq 10^{-47}$  GeV<sup>4</sup>, provided  $\rho_\Lambda \simeq \rho_\Lambda^{ZPE}$ .

Although the estimate for the CC energy density obtained by conventional means [Eq. (3)] and supplemented by the holographic restriction [Eq. (2)] matches its presently observed value, it has been pointed out recently [11] that the dark energy equation of state is strongly disfavored by the observational data. Namely, assuming that ordinary energy-momentum tensor associated to matter and radiation is conserved, one easily finds using Friedman equation that the CC and ordinary matter scale identically,  $\rho_\Lambda \sim \rho_m$ . Hence, dark energy scales as pressureless matter ( $\omega \equiv p/\rho = 0$ ), while the most recent data indicate  $-1.38 \leq \omega \leq -0.82$  at 95% confidence level (see e. g. [12]). A proposed remedy [13, 14] of the problem relies on the event horizon as a new choice for the infrared cutoff  $L$ . In this case the present equation of state improves to  $\omega = -0.90$ ; the model is, however, unable to address the cosmic ‘coincidence’ problem [15].

In the present paper, we point out that taking the ordinary energy-momentum tensor as individually quantity conserved may be compatible with the possibility that  $\Lambda$  be a function of the cosmological time [as indicated by holography through Eq. (4)] only in two special cases. In the first case, introduction of some additional terms in the Einstein field equations is necessary; in a tensor-scalar theory of gravity, for example, such additional terms are functions of a new scalar field. Although this point of view might be welcomed in the light of the quintessence proposal, one should however bear in mind that here we deal all the time with variable but ‘true’ CC [Eq. (3)] with  $\omega_\Lambda = -1$ . In another special case, the scaling behavior of  $\rho_\Lambda$  may be quite different from the law  $\rho_\Lambda \sim L^{-2}$  [Eq. (4) with  $G_N$  constant], which was the basic assumption in [11, 13, 14].

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<sup>1</sup> It has been argued in [10] that the quartic divergence is actually absent in  $\rho_\Lambda^{ZPE}$  as a consequence of the relativistic invariance which requires  $\rho_\Lambda = -p_\Lambda$ , where  $p_\Lambda$  is the vacuum pressure. But even so, this has no influence on the present discussion and the form of Eq. (4).

Indeed, considering the Einstein field equation

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu} , \quad (5)$$

where  $G_{\mu\nu} = R_{\mu\nu} - Rg_{\mu\nu}/2$  is the Einstein tensor and  $T_{\mu\nu}$  is the energy-momentum tensor of ordinary matter and radiation, one sees by Bianchi identities that when the energy-momentum tensor is conserved ( $\nabla^\mu T_{\mu\nu} = 0$ ), it follows necessarily that  $\Lambda = \text{const.}$ . We stress that there are actually three ways to accommodate the running of the CC with the cosmological time,  $\Lambda = \Lambda(t)$ , with the Einstein field equation. The most obvious way is to shift the CC to the right-hand side of Eq. (5), and to interpret the total energy-momentum tensor  $\tilde{T}_{\mu\nu} \equiv T_{\mu\nu} - \frac{\Lambda(t)}{8\pi G} g_{\mu\nu}$  as a part of the matter content of the universe. By requiring the local energy-conservation law, ( $\nabla^\mu \tilde{T}_{\mu\nu} = 0$ ), we obtain

$$\dot{\rho}_\Lambda + \dot{\rho}_m + 3H\rho_m(1 + \omega) = 0 . \quad (6)$$

We note that the time evolution of  $\rho_\Lambda$  and  $\rho_m$  is coupled in (6). The equation of state for ordinary matter and  $\Lambda$  in (6) is  $\omega$  and -1 respectively. It is important to note that both  $\rho_\Lambda$  and  $\rho_m$  do not evolve according to the  $\omega$ -parameter from their equations of states. The important implication of Eq. (6) is that there exists an interaction between matter and the CC which causes a continuous transfer of energy from matter to the CC and vice versa, depending on the sign of the interaction term. The interaction between the two components may be defined as (for pressureless matter with  $\omega = 0$ )

$$\begin{aligned} \dot{\rho}_m + 3H\rho_m &= X , \\ \dot{\rho}_\Lambda &= -X , \end{aligned} \quad (7)$$

where the coupling term  $X$  is to be determined below. Taking for definiteness  $\rho_\Lambda = CM_P^2 H^2$ , we obtain with the aid of the Friedman equation for the flat-space case that

$$\begin{aligned} \rho_\Lambda \sim \rho_m &\sim a^{-3(1-\frac{8\pi C}{3})} , \\ X &= 8\pi CH\rho_m , \end{aligned} \quad (8)$$

thus showing a rather different result for  $\rho_m$  than the standard behavior  $a^{-3}$ . In addition, for curved universes the scaling  $\rho_m \sim \rho_\Lambda$  is absent, a welcome feature for the problem of structure formation [16].

A conventional field-theoretical model with the CC running fully in accordance with the holographic requirement (4) has been put forward recently [17]. It was based on the

observation [18] that even a ‘true’ CC in such theories cannot be fixed to any definite constant (including zero) owing to the renormalization-group (RG) running effects. The variation of the CC arises solely from the particle field fluctuations, without introducing any quintessence-like scalar fields. Particle contributions to the RG running of  $\Lambda$  due to vacuum fluctuations of massive fields have been properly derived in [19], with a somewhat peculiar outcome that more massive fields do play a dominant role in the running at any scale. Assuming some kind of merging of quantum field theory with quantum gravity (or string theory) near the Planck scale, one may explore the possibility that the heaviest degree of freedom may be associated to particles having masses just below the Planck scale (or the effective value of mass in that regime may be due to multiplicities of such particles). While in the perturbative framework of the model the running of the Newton’s constant is negligible [20], the scaling of the CC is just of the form as above,  $\rho_\Lambda = CM_P^2 H^2$ . It was shown (second Ref. in [17]) that for  $C \sim 10^{-1} - 10^{-2}$  (safely within the holographic bound) the model is compatible with all observational data and can be tested in future Type Ia supernovae experiments [21]. Moreover, the ‘coincidence’ problem is simply understood by noting that  $(\rho_\Lambda^0)^{1/4} \sim \sqrt{M_P H_0}$  is given by the geometrical mean of the largest and the smallness scale in the universe today. Hence, we see that the holographic relation [Eq. (4) with  $G_N$  constant] is consistent with current cosmological observations and does not suffer from the ‘coincidence’ problem.

Let us also mention that from other considerations in line with the holographic conjecture, the same law for  $\rho_\Lambda$  has been recently reached in [22]. Also, there is a recent paper [23] reaching similar conclusions from general arguments in Quantum Field Theory.

As already stated, if one ignores the presence of additional light scalars (which do not influence the present discussion anyway), the variable CC can be achieved also by promoting the Newton’s constant to a time-dependent quantity. In this case the variation of the CC can be maintained even if the energy-momentum tensor for ordinary matter stays conserved. In this particular case,  $\nabla^\mu(G(t)\tilde{T}_{\mu\nu}) = 0$  implies the equation of continuity to be

$$\dot{G}(\rho_\Lambda + \rho_m) + G\dot{\rho}_\Lambda = 0 . \quad (9)$$

Hence the scaling of  $\rho_\Lambda$  in (9) is coupled with the scaling of  $G$ .<sup>2</sup>

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<sup>2</sup> The more general case, of course, would have both  $X \neq 0$  and  $\dot{G} \neq 0$ , but it is not *a priori* clear whether such a model can be made compatible with the holographic relation (4) as well as the observational data.

A complementary approach to that of the model [17] for the RG evolution of the CC [also obeying (9)] has been put forward in [24]. Now, the RG running is due to non-perturbative quantum gravity effects and a hypothesis of the existence of an IR attractive RG fixed point. In contrast to the model [17], a prominent scaling behavior of the gravitational constant was found in [24]. The behavior for the spatially flat universe was found to be  $\rho_\Lambda \sim H^4$ ,  $G_N \sim H^{-2}$ . Again, the model might explain the data from recent cosmological observations without introducing a quintessence-like scalar field. Moreover, the model does predict that near the fixed point  $\rho_m = \rho_\Lambda$ , which is quite close to the values favored by recent observations. It is therefore up to the fixed point structure to provide for the mysterious approximate equality of  $\rho_m$  and  $\rho_\Lambda$  at present (the ‘coincidence’ problem). Hence, we see that our ‘generalized’ holographic relation (4), where now both  $\rho_\Lambda$  and  $G_N$  are varying, can be also made consistent with the present cosmological data and may alleviate the cosmic ‘coincidence’ problem.

To summarize, we have shown that the holographic ideas discussed in the present paper yield the behavior of the CC which is consistent with current observations. This is true even for a ‘true’ CC with the equation of state being precisely -1 and with the Hubble distance as a most natural choice for the IR cutoff. We have noted that the presence of quintessence-like scalar fields is redundant in the present approach and not required for the consistency with observational data. Our conclusion is that the scaling of the CC stemming from holography unavoidably implies either a nonvanishing coupling of the CC with dark matter or a time-dependent gravitational constant.

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- [1] T. Banks, hep-th/9601152; hep-th/0007146.
  - [2] A. Cohen, D. Kaplan, and A. Nelson, Phys. Rev. Lett. 82, 4971 (1999).
  - [3] P. Horava and D. Minic, Phys. Rev. Lett. 85, 1611 (2000).
  - [4] S. Thomas, Phys. Rev. Lett. 89, 081301 (2002).
  - [5] S. Weinberg, Rev. Mod. Phys. 61, 1 (1989).

- [6] For an excellent review of the holographic principle see, R. Bousso, *Rev. Mod. Phys.* 74, 825 (2002).
- [7] G. 't Hooft, gr-qc/9310026.
- [8] L. Susskind, *J. Math. Phys.* 36, 6377 (1995).
- [9] J. D. Bekenstein, *Phys. Rev. D* 7, 2333 (1973); *Phys. Rev. D* 23, 287 (1981).
- [10] E. Kh. Akhmedov, hep-th/0204048.
- [11] S. D. H. Hsu, hep-th/0403052.
- [12] A. Melchiorri, L. Mersini, C. Odman, and M. Trodden, *Phys. Rev. D* 68, 43509 (2003).
- [13] M. Li, hep-th/0403127.
- [14] Q. G. Huang and Y. Gong, astro-ph/0403590.
- [15] P. J. Steinhardt, in "Critical Problems in Physics", edited by V. L. Fitch and Dr. R. Marlow (Princeton University Press, Princeton, N. Y., 1997).
- [16] M. S. Turner, *Int. J. Mod. Phys. A* 17S1, 180 (2002); *Int. J. Mod. Phys. A* 17, 3446 (2002).
- [17] I. L. Shapiro and J. Sola, *Phys. Lett. B* 574, 149 (2003); C. E. Espana-Bonet, P. Ruiz-Lapuente, I. L. Shapiro, and J. Sola, *JCAP* 0402, 006 (2004).
- [18] I. L. Shapiro and J. Sola, *Phys. Lett. B* 475, 236 (2000).
- [19] A. Babic, B. Guberina, R. Horvat, and H. Stefancic, *Phys. Rev. D* 65, 085002 (2002); B. Guberina, R. Horvat, and H. Stefancic, *Phys. Rev. D* 67, 083001 (2003).
- [20] I. L. Shapiro and J. Sola, *JHEP* 0202, 006 (2002).
- [21] See e. g., <http://snap.lbl.gov/>.
- [22] S. Carniero and J. A. S. Lima, gr-qc/0405141.
- [23] T. Padmanabhan, hep-th/0406060.
- [24] A. Bonnano and M. Reuter, *Phys. Lett. B* 527, 9 (2002).